Journal of Alloys and Compounds, 195 (1993) 615–618
JALCOM 7541

A model for low-field flux penetration in disordered superconductors

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Abstract

The experimental scaling properties of the first flux penetration in a disordered superconductor (critical exponent $\varphi = 2.7$) are derived from a granular fractal model. It assumes localisation of the order parameter in randomly coupled regions and a temperature dependent cutoff in the couplings. Screening currents circulate in percolation fractal clusters with temperature dependent percolation parameter. Their description requires the heuristic transposition of classical calculations of first penetration fields in superconductors into fractal space.

1. Introduction

Measurements of the diamagnetic response of $YBaCuO_{6.7}F_x$ $(0 \le x \le 0.14)$ ceramics [1] at very low fields (from 1 m Oe to 1 Oe), show that the field of first flux penetration follows a law $H_1(T,x) = H_1(0,x)\varepsilon^{\varphi}$ with $\varepsilon = |1-T/T_c|$ and $\varphi = 2.7 \pm 0.2$, independently of the amount of fluorine. Furthermore, a universal behaviour is observed above H_1, where flux motion in the sample is expected to be irreversible.

Our purpose here is to show that the critical behaviour described by the exponent φ and its value, are necessary consequences of disorder in the samples. Disorder also provides a hint as to the origin of universal irreversible behaviour.

The existence of a critical field $H_{C1} = \Phi_0 \ln(\lambda_s/\xi_s)/4\pi\lambda_s^2\,\mu_0$ for the transition between the Meissner and the single-vortex state in type II superconductors, as well as that of a barrier field $H_s = \Phi_0/4\pi\lambda_s\,\xi_s$ opposing the passage of a vortex across a surface, have been known for a long time [2]. Here λ_s and ξ_s are the Ginzburg-Landau penetration depth and coherence length, respectively ; they both vary like $\varepsilon^{-1/2}$, which precludes the fitting of experimental results by any of the two fields.

The diamagnetic response of $YBa_2Cu_3O_{6.7}F_x$ samples obeys the same law in the granular sample with x = 0 as in the others where only compositional disorder varies with x [3]. We then assume that in all cases there is a localisation of the superconducting order parameter in randomly coupled regions of size $a >> \xi_s(T)$. This allows us to apply and extend a model [4,5] which has successfully described properties of weakly-coupled granular superconductors, in particular the penetration depth $\lambda(T) \sim \varepsilon^{-\beta}$ $(\beta = 0.7 \pm 0.1)$ due to Josephson-like intergranular screening currents. The main ideas of this model are : (i) the weakest couplings $J << k_BT$ have little influence on the critical behaviour near the coherence temperature T_c ; only couplings above a cutoff $J^*(T)$ are relevant, which defines a percolation parameter $p[J^*(T)] = \text{Prob}(J > J^*)$; (ii) percolation clusters and thermally correlated regions (grain-to-grain correlation length $\xi(T)$) tend to coincide, i.e. the percolation correlation length $\xi_P \sim \xi(T) \sim \varepsilon^{-\nu}$ and $|p/p_c - 1| \sim \varepsilon^{\nu/\nu_P}$, where ν and ν_P are thermal and percolation exponents, respectively ; thus p(T) is independent of the particular distribution of couplings in the sample ; (iii) thermal and percolation exponents are related, and in particular $\nu = 2\nu_P/3\beta_P \cong 1.35$, in very good agreement with the experimental results [6] ; (iv) intragranular screening currents are taken into account by simply assuming that couplings $J < J^*(T)$ form a "medium" with permeability $\mu(\lambda_s/a) < \mu_o$ in which the percolation clusters are imbedded.

One of the essential features of these clusters is their fractal dimension. The question arises : how to describe the fields observed in euclidean (d-dimensional) space, actually generated by currents circulating in fractal (D-dimensional, D < d) space ? We answer this question heuristically in what follows, and transpose the classical calculation of H_{C1} and H_s to the infinite cluster.

2. Currents and fields in fractal clusters

We remark that D-dimensional clusters occupying a volume L^d contain $(L/a)^D$ elements (junctions, grains...). Here a is a microscopic unit of length. If a fractal being existed, he would measure a density $(L/a)^D/L^D = a^{-D} = \text{const}$, while euclidean observers would find $(L/a)^D/L^d = a^{-d}\,(L/a)^{D-d}$.

0925-8388/93/$06.00

2.1 Transformation of densities

The preceding scheme applies to all extensive quantities. Euclideans and Fractalians see the same *total* free energy F_1, magnetic flux Φ, current I, ..., but different vortex energies per unit length f_1, flux $(\vec{b})$ and current $(\vec{j})$ densities, etc. More precisely, one can show that if $f(\vec{r})$ is a density defined in a fractal space the averages

$$f = \langle f(\vec{r})\rangle_d, \quad f_D = \langle f(\vec{r})\rangle_D, \quad f = Wf_D \tag{1}$$

taken on d- and D-dimensional spaces,respectively, are related by the dilution factor

$$W(L) = \begin{cases} (a/L)^{d-D} \text{ if } a \le L < \xi \\ (a/\xi)^{d-D} \text{ if } L > \xi \end{cases} \tag{2}$$

Notice that the intersections [7] of a segment of length L or a surface of area L^2 with the fractal have dimensions D-2 or D-1, respectively, and fractal "length" LW(L) or "area" $L^2W(L)$. We can thus form Table 1, containing all quantities necessary for our calculations. They will usually appear under an integral sign, so the value of W will be determined by the domain of integration. The mesoscopic flux density $\vec{b}$ is related to the induction by $\vec{B} = \langle \vec{b} \rangle_d$.

2.2 Flux penetration

Table 1 provides the information necessary to formulate a physical problem in D dimensions and to solve it in euclidean space. This correspondence insures in particular that the operator ∇_D satisfies *formally* usual vector identities. These assertions are usefully illustrated by the calculation of the energy F_1 of an isolated Josephson-like vortex line over a length L in an infinite cluster. In the London approximation one has to minimize the sum of magnetic and kinetic energies

$$\begin{aligned} F_1(L) &= \int \left[b_D^2 + \lambda_D^2 \left| \nabla_D \Lambda \vec{b}_D \right|^2 \right] \frac{d^D\vec{r}}{2\mu_D} \\ &= \int \left[b^2 + \lambda^2 \left| \nabla \Lambda \vec{b} \right|^2 \right] \frac{d^D\vec{r}}{2\mu W}, \end{aligned} \tag{3}$$

which leads to London equations for both $\vec{b}_D$ and $\vec{b}$. Using these equations and well-known vector identities, one obtains

$$\begin{aligned} F_1(L) &= \int d^D\vec{r}\nabla_D.(\vec{b}_D \Lambda \nabla_D \Lambda \vec{b}_D)\lambda_D^2/2\mu_D \\ &= \int d^{D-1}\,\vec{r}.(\vec{b}\Lambda\nabla\Lambda\vec{b})\lambda^2/2\mu W \\ &= \frac{\Phi_0^2}{4\pi\lambda^2\mu W(a)} \ln\frac{\lambda}{a} \int d^{D-2}\vec{r} \end{aligned} \tag{4}$$

The first integration uses the divergence theorem after replacing ∇_D by $W^{-1}\nabla$. The second integration is performed around a cylinder of radius a (the vortex is of Josephson type and therefore has no normal core), parallel to the field. This integration determines the value W = W(a) = 1 in the denominator. The expression before the integral in the last step results from the vortex solution of the London equation, $b = \Phi_0 \ln(\lambda/r)/2\pi\lambda^2$, for $a < r < \lambda$. Finally, the integral itself is the length of the intersection of the cylinder axis with the fractal, i.e. LW(L). For $L \gg \xi(T)$

$$H_1 = \frac{F_1}{L\Phi_0} = \left(\frac{a}{\xi}\right)^{d-D} \frac{\Phi_0}{4\pi\lambda^2\mu} \ln\frac{\lambda}{a} \tag{5}$$

is the critical field for vortex penetration [2] in the volume of the superconductor.

Table 1. Extensive quantities (first row), their d-dimensional densities (third row) and the relation of the latter with their D-dimensional counterparts (fourth row). The last column concerns the penetration depth.

Invariant	F_1	Φ	I	$L = \Phi/I$	E_m		
d	1	2	2	1	3	1,2,3	1
X	f_1	$\vec{b}$	$\vec{j} = \frac{\nabla\Lambda\vec{b}}{\mu}$	μ	$b^2/2\mu$	∇	λ
$X = X(X_D)$	Wf_1	$W\vec{b}_D$	$\frac{W\nabla_D\Lambda\vec{b}_D}{\mu_D}$	$W\mu_D$	$\frac{Wb_D^2}{2\mu_D}$	$W\nabla_D$	λ_D/W

The surface barrier field H_s is obtained by applying similar rules to the calculation of the Gibbs potential, $G_1 = F_1 - \int \vec{b}.\vec{H}d^d\vec{r}$. Besides the vortex solution, the total field is formed by an image anti-vortex outside the cluster and the regular solution with boundary condition $b = \mu H$ on the surface of the cluster. Attractive and repulsive interactions result in a barrier energy which depends on the distance x_1 between the vortex and the surface. The condition of a maximum of G establishes a relation $H(x_1)$. We skip the details of this classical calculation [2], which results in the fractal case in a maximum value of H when $x_1 \cong \xi(T)$:

$$H_{sl} = \frac{\Phi_0}{2\pi\lambda\xi\mu}\frac{W(\xi)}{1+W(\xi)}. \tag{6}$$

The condition $x_1 \cong \xi$ is equivalent to making critical the current circulating between the vortex and the surface. Indeed, the inter-granular phase gradient due to the current circulating far from the surface is of order $2\pi/2\pi\lambda = 1/\lambda$, because the phase changes by 2π around a vortex. The same current must cross the segment of length x_1 on the other side of the vortex, so the phase gradient increases by a factor of about λ/x_1, to become $1/x_1$. In fact, the current becomes critical when the phase gradient is of order $1/\xi(T)$ [4,5]. In a way, we have applied Silsbee's rule [8] to disordered superconductors.

3. Discussion

In the case of conventional superconductors the fields H_{c1} and H_s have the same temperature dependence. This seems not to be the case of H_1 and H_{s1} given by equations (5) and (6). In effect,

$$H_1 \approx \varepsilon^{(d-D)\nu+2\beta}, \tag{7}$$

$$H_{sl} \approx \varepsilon^{\beta+(d+1-D)\nu}. \tag{8}$$

The question arises of which field is actually measured or rather, what is the relevant fractal dimension. If the two exponents were significantly different a crossover between the two behaviours would be expected. It is not observed experimentally, at least in the temperature interval explored [1]. So it is conceivable that different dimensionalities are relevant in the two cases. Actually screening currents can only circulate in closed loops, which would eliminate "dead ends" in the fractal and leave the backbone dimension $D_b \cong d-1 = 2$ [9]. This may be the situation well inside the cluster, where Eq.(7) is relevant. But the surface field (8) depends on the interaction between the vortex and its image, and here the dead ends will be influent on the strength, shape and even number of images. It is suggestive that if D_b is written instead of D in Eq.(7), and the exponents in Eq.(7) and (8) are made equal, taking into account the scaling relation $2\beta = (d-2+\eta)\nu$ with $\eta << 1$, one obtains $D \cong D_b + 0.5$. This is very close of the percolation dimension $D = d-\beta_P/\nu_P = 2.53$. When the known values of D_b, D, β and ν are replaced, one obtains $\varphi \cong 2.7$ in both cases.

Another characteristic field should have a similar temperature dependence. When $H \geq H_2 = \Phi_0/\mu\xi^2$, the distance between vortices becomes smaller than the intergranular (percolation) correlation length. So to speak, the vortices start seeing the fractal structure in the clusters. Independently of a more detailed description, we can expect this field to be associated with a particular region in the curve of the diamagnetic response versus field, probably the approach to saturation. Since this curve was shown in Ref. [1] to be universal for all temperatures when plotted as a function of $H/H_1(T)$, a necessary condition for such universality is that $H_2(T)/H_1(T)$ be independent of temperature. This is numerically the case : $H_2 \sim \varepsilon^{2\nu}$, and $2\nu \cong \varphi \cong 2.7$.

References

1. J.P. Burin, Y. Fouad, A. Raboutou, P. Peyral, C. Lebeau, J. Rosenblatt, M. Mokhtari, O. Peña and C. Perrin, paper in this volume.
2. See for example P.G. de Gennes, Superconductivity of Metals and Alloys, W.A. Benjamin, New York, 1966 ; D. Saint-James, E.J. Thomas and G. Sarma, Type II Superconductivity, Pergamon Press, Oxford, 1969.
3. C. Perrin, A. Dinia, O. Peña, M. Sergent, P. Burlet and J. Rossat-Mignod, Solid State Commun. 76 (1990) 401.
4. J. Rosenblatt, Phys. Rev. B 28 (1983) 5316 ; J. Rosenblatt in Percolation, Localisation and Superconductivity, A.M. Goldman and S.A. Wolf (Eds), NATO ASI series B, Physics, Plenum Press, New York, Vol 109, p. 431, 1984.
5. J. Rosenblatt, C. Lebeau, P. Peyral and A. Raboutou in Advances in the Physics of Condensed Matter : Josephson Effects, Achievements and Trends, A. Barone (Ed.) World Scientific, Singapore, p. 320, 1986.
6. J. Rosenblatt, P. Peyral, A. Raboutou and C. Lebeau, Physica B 152 (1988) 100.
7. B.B. Mandelbrot, The Fractal Geometry of Nature, W.H. Freeman and Co., San Francisco, 1982.
8. F.B. Silsbee, J. Wash, Acad. Sci. 6 (1916) 597.
9. Y. Gefen, A. Aharony, B.B. Mandelbrot and S. Kirkpatrick, Phys. Rev. Lett. 47 (1981) 1771.